

# A Theory of Changes and the Fundamental Plane

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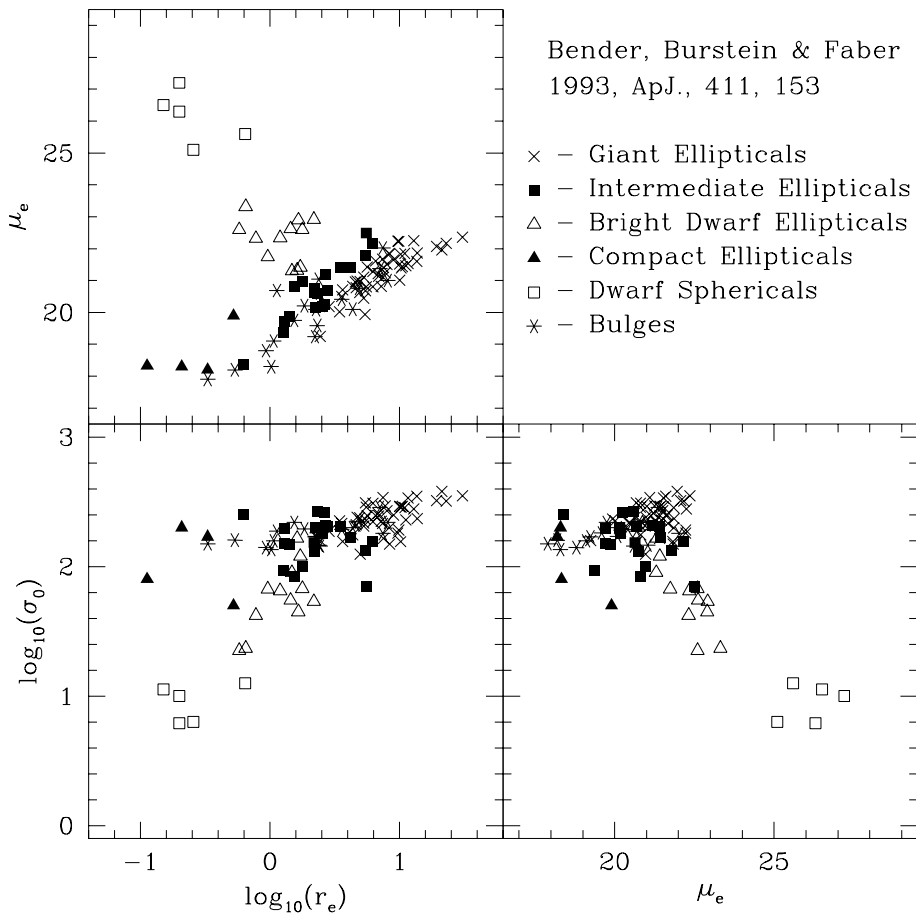
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**Abstract:** Elliptical galaxies have been observed to cluster near a ribbon along a two-dimensional plane (the Fundamental Plane of Elliptical Galaxies) in the three dimensional space of effective radius, effective surface brightness and central velocity dispersion. This observed clustering holds for galaxies spanning several orders of magnitude in size and total brightness and residing in a wide variety of environments. In order to understand the Fundamental Plane (FP), it is important to know both why it should arise, and how it will evolve. This contribution addresses the second of these, how the FP is likely to change with time. In that context, we consider the evolution due to galaxy–galaxy interactions, starting with arguments based upon the virial theorem, and a discussion of the importance of understanding the mass-to-light ratio as a function of position in the galaxies. The basic result of this argument is that most of any global change in a galaxy due to a transformation (of whatever cause) is mostly projected along the Fundamental Plane. A secondary point is that if the change includes a substantial change in the mass-to-light ratio, a galaxy can move across the FP, which could help explain the differences in observational properties between the regular and dwarf galaxies.

## 1. Introduction

The clustering of elliptical galaxies along a two dimensional manifold, the Fundamental Plane, in a 3-space of effective radius ( $r_e$ ), effective surface brightness ( $\Sigma_e$ ,  $\mu_e = -2.5 \log \Sigma_e$ ) and central velocity dispersion ( $\sigma_0$ ) (Djorgovski & Davis 1987, Dressler et al. 1987, Bender, Burstein & Faber 1992, 1993, Guzmán, Lucey & Bower 1993, Pahre, 1996; see figure 1) requires an explanation. An additional point that has been noted before (e.g. Kormendy 1985, Bender et al. 1992,1993) is that the dwarf galaxies appear to behave differently from the larger, regular elliptical galaxies.

The fact that the galaxies lie on a plane has been explained by assuming they are in virial equilibrium (e.g. Faber et al 1987, Bender et al 1992), and, following the work of Poveda (1958), the distribution of mass has been presumed to be directly proportional to the distribution of light, so that a plane in mass becomes a plane in light. While a promising first approximation, this left open the questions of why the galaxies appear to lie along a “ribbon” on the manifold, why the dwarf galaxies appear discrepant, and why there was some divergence from the plane defined by virial equilibrium.



**Figure 1.** Observed properties of a range of elliptical systems (data are from table 1 of Bender, Burstein & Faber 1993). The dwarf galaxies ( $\square$  &  $\triangle$ ) are thought to have formed differently from the larger systems and/or may have evolved differently, though through similar processes.

This does not address the question of how a galaxy should appear to move in this space as it evolves. For example, Faber (1973) showed how tidal stripping of the outer portions of an elliptical galaxy could explain the existence of several elliptical galaxies with high surface brightness and low luminosity. In this case, assuming that the outer portions of the galaxy simply vanish, the inner portion comes to some new dynamical equilibrium, which can be characterized by the virial theorem. Levine & Aguilar (1996) expanded upon the virial theorem argument to allow for the possibility of a mass-to-light ratio that is a function of position and showed how changes in the underlying total mass and binding energy of a galaxy due to some interaction can be related to the galaxy's observable properties.

In this contribution, I generalize the earlier argument of Levine & Aguilar (1996) and show how a better understanding of the fates of luminous and dark

matter as a result of interactions is crucial to understanding the motions of galaxies in the space of observable properties. This can also help us to understand why “regular” elliptical galaxies exhibit a FP and why dwarf galaxies seem so at odds with their larger cousins. First I will discuss how to convert light to mass, and then fold that into the argument of virial equilibrium, and wrap up with a discussion of the projected dispersions in observable properties.

## 2. Light to Mass

When we observe a galaxy, by definition, we are observing LIGHT. If we have taken spectra, then that light can be transformed into a measure of the underlying potential of the galaxy through the velocity dispersion. We have no other direct measure of the underlying MASS of the galaxy, unless we have some way to convert measurements of light into mass. The real dynamics is all based upon the distribution of mass, including the assumption of virial equilibrium. The simplest and most often used assumption has been that the underlying mass is in direct and constant proportion to the observed luminosity. In this case, the results predicated upon the dynamical argument of virial equilibrium translate directly into observed light profiles.

Binney (1982b), Hjorth & Madsen (1991, 1995) and Spergel & Hernquist (1992) have all noted that the natural product of an incomplete violent relaxation process is an  $r^{1/4}$  type profile. Aguilar & White (1986) found empirically in their N-body simulations of hyperbolic encounters that the radial density profile that resulted from upsetting either a de Vaucouleurs or a King model obeyed an  $r^{1/4}$  law. Barnes and collaborators have found similar results in merging encounters (see Barnes 1994 for an extended review of the work). These are arguments for why the underlying mass profile should follow an  $r^{1/4}$  law. As Hjorth & Madsen (1991) note in the very beginning of their work, this is then consistent with the observed profiles *if* the mass-to-light ratio is constant over the galaxy. This constancy is a sufficient, but not a necessary condition for linking mass and light.

We are not limited to a constant mass-to-light ratio even if one takes the observed robust result that the light in most elliptical galaxies obeys an  $r^{1/4}$  law, and the dynamical argument that the mass should too. In the more general case where mass and light each satisfy possibly distinct de Vaucouleurs laws, the ratio of surface density  $[\varsigma(r)]$  to surface brightness  $[\Sigma(r)]$ ,  $\alpha(r) = \varsigma(r)/\Sigma(r)$  is an  $r^{1/4}$  law function

$$\alpha(r) = \alpha_{r=0} \exp \left[ -c \left( \frac{r}{\alpha_e} \right)^{1/4} \right] \quad (1)$$

where  $c$  is a constant, and the “effective M/L radius”  $\alpha_e$  and  $\alpha_{r=0}$  are defined as

$$\frac{1}{\alpha_e^{1/4}} \equiv \left[ \frac{1}{r_g^{1/4}} - \frac{1}{r_e^{1/4}} \right] , \quad \text{and} \quad \alpha_{r=0} = \frac{\varsigma(r=0)}{\Sigma(r=0)} . \quad (2)$$

$r_g$  and  $r_e$  are the effective radii for the mass and light profiles respectively. In this manner, we can connect the observed light with the underlying mass, and we end up with a more flexible description of  $\alpha(r)$ . Obviously if  $r_g = r_e$  then  $\alpha$  is a constant and equal to the global  $M/L$ . This is not to imply that this is necessarily the correct connection, merely that the connection need not be a constant for theory and observation to meet.

We need a better understanding of the variation of mass-to-light ratio as a function of position. With that, we can then construct a more complete representation of the distribution of galaxy mass, check the virial assumption and work towards a better understanding of how mass-to-light *changes* over the course of galaxy-galaxy interactions.

### 3. Change in Mass to Change in Light

In the case where an elliptical galaxy is changed under a transformation that satisfies the following properties before and after the change

- (i) *the galaxy is in virial equilibrium,*
- (ii) *the mass and the light profiles have the same 2-parameter functional form,*
- (iii) *the functional forms of the radial surface profiles do not change and,*
- (iv) *there exists a radius for which the local mass-to-light ratio is constant over an encounter*<sup>1</sup>

we can relate the changes in mass  $M$ , binding energy  $E$ , global mass-to-light ratio  $\langle\alpha\rangle$ , and  $\alpha_{r=0}$  to changes in the observed galaxy properties.

The first criterion can be justified on theoretical grounds, since the time scales for dynamical evolution for the inner regions of elliptical galaxies are quite short compared to their lifetimes. On observational grounds the smooth distribution of light that is prevalent in ellipticals argues for a least a quasi-equilibrium state (see recent reviews by Bertin & Stiavelli 1993, de Zeeuw & Franx 1991 and Binney 1982a).

The argument for a two parameter form (for the luminous profile) is based upon the presumption that we can characterize an elliptical galaxy solely by 2 parameters, e.g. total mass and internal binding energy. Djorgovski (1987) points out that the residuals in the correlations of the various observed properties seen in elliptical galaxies indicate that only two parameters are needed to describe their global properties. That the mass profile is of the same form is an assumption based upon the arguments of the previous section and clearly limited by our knowledge of the true variation of mass with light.

In observations of clearly interacting spiral galaxies (e.g. Schweizer 1982), in the centers, an  $r^{1/4}$  profile can be nicely fitted to the radial surface brightness, indicating the robustness of the  $r^{1/4}$  profile as an outcome of strong interactions between galaxies, even with non-elliptical progenitors. This meshes well with the

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<sup>1</sup> The results given in Levine & Aguilar (1996), are for the case when the constant point is the center,  $r = 0$ .

previously noted theoretical reasons for expecting a similar profile. Assumption (iv) helps to restrict the solution space by a dimension, and especially in the cases of moderate encounters, is likely to be true near or at the center of the galaxy, since we would expect only a quite strong transformation to have a large effect all the way in as far as the center.

For a given variable  $x$ , we define the change in  $x$  as  $\Delta x \equiv x_{\text{final}}/x_{\text{initial}}$ . The changes in the observable galaxy properties are then related to the changes in the global properties in the following manner:

$$\begin{aligned}\log(\Delta r_e) &= 2\log(\Delta M) - \log(\Delta E) - \frac{1}{2}\log(\Delta\langle\alpha\rangle) + \frac{1}{2}\log(\Delta\alpha_{r=0}) \\ \log(\Delta\sigma_0) &= \frac{1}{2}[-\log(\Delta M) + \log(\Delta E)] \\ \log(\Delta\Sigma_e) &= -3\log(\Delta M) + 2\log(\Delta E) - \log(\Delta\langle\alpha\rangle) \quad .\end{aligned}\tag{3}$$

#### 4. Interaction to Change in $M$ , $E$ , $\langle\alpha\rangle$ and $\alpha_{r=0}$

In order to be able to decipher the effects of tidal interactions upon the observed properties of elliptical galaxies, we have undertaken a series of numerical  $N$ -body simulations to determine empirically the relationship between the parameters describing a tidal interaction and the changes in global properties ( $M$ ,  $E$  etc) (Levine & Aguilar 1997). These are in essence interaction cross-sections.

To keep the project to a reasonable size, and still maintain decent resolution in each simulation, we have limited ourselves to exploring a range in galaxy mass ratio ( $M_1/M_2 = 1, 2, 4, 8$ ), impact velocity ( $v_\infty = [1/2, 1, 2, 4] \times \sigma_0$ ) and impact parameter ( $p_\infty = [0, 1, 2, 4, 8, 16] \times r_{g,1}$ ). In all a set of 96 simulations have been run, covering both merging and hyperbolic encounters. Vergne & Muzzio (1995) have also performed a large series of simulations designed to explore the interaction parameter space, but they lacked the necessary numerical resolution that we have here. Otherwise, almost all such work has been done using either only equal mass galaxies or a rigid perturber or has only considered a very few interactions (e.g. Aguilar & White 1985, Capelato, de Carvalho & Carlberg 1995). The main thrust has often been to model a particular interaction in more detail, rather than to map out the parameter space (which admittedly is a very computationally expensive proposition).

These cross-sections can then be used for (among many other things) a Monte-Carlo experiment similar to that used by Levine & Aguilar (1996), where they used an older set of equal mass interaction cross-sections to try to compare the effect of tidal interactions with the observed spread in galaxy properties.

## 5. Discussion and Speculation

Assuming that there is a way to convert interaction (or transformation) parameters into changes in observable galaxy properties, we can now ask, how does an ensemble of galaxies change under interactions that satisfy the criteria given above?

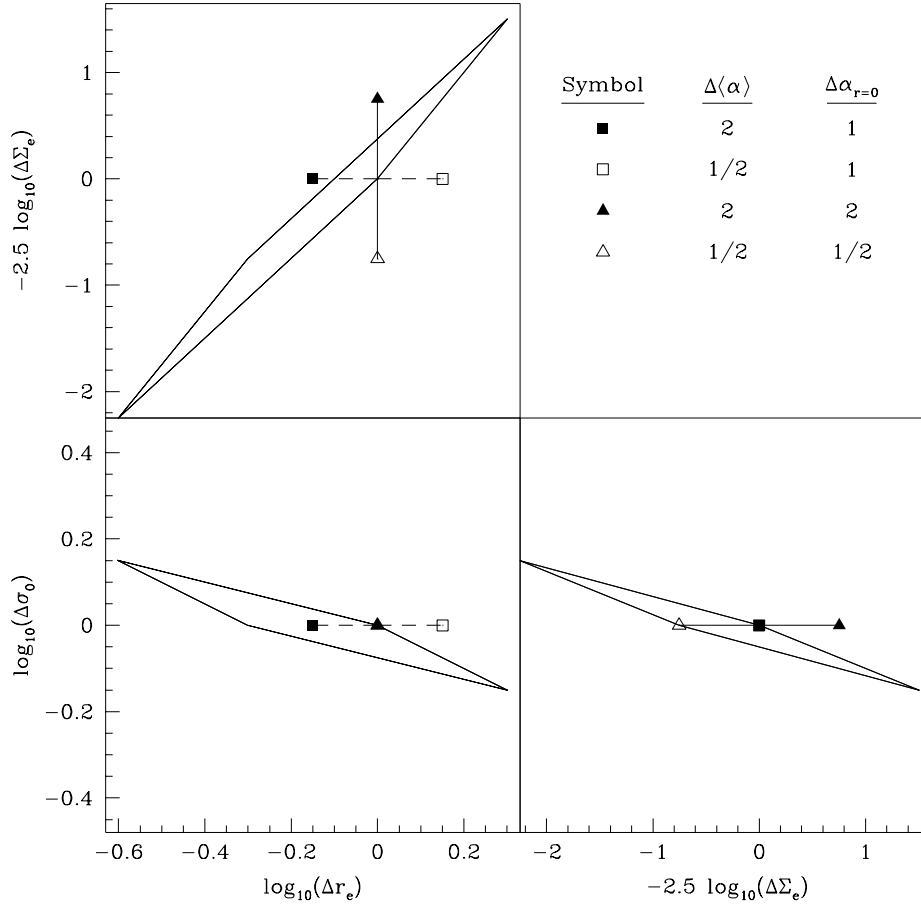
For large galaxies subject to moderate non-merging encounters, we find that the change in total mass and binding energy is fairly small. In addition, we expect that the change in central surface brightness and central surface density will be small to non-existent ( $\Delta\alpha_{r=0} \approx 1$ ). Any differential change of luminous versus dark matter will show up as a change in the overall mass-to-light ratio. The parallelograms in figure 2 show the bounding box of change in observable parameters given a change in  $M$  and  $E$  equal to a factor of  $1/2$ . The dashed line extending out from the point (0,0) shows how the parallelogram would be displaced if  $\Delta\langle\alpha\rangle = 1/2$  and 2 and  $\Delta\alpha_{r=0} = 1$ .

For dwarf galaxies, the change in  $M$  and  $E$  can be quite substantial. Similarly, because the encounters can affect these galaxies so dramatically, we suppose that any difference in the efficiency of change with respect to light and dark mass will be felt all the way into the center, and so  $\Delta\alpha_{r=0} \approx \Delta\langle\alpha\rangle$ . The solid line shows how the parallelogram of change would be displaced if  $\Delta\langle\alpha\rangle = \Delta\alpha_{r=0} = 1/2$  and 2. This is only a guide to the accessible regions of the observable parameter space. Based on the results of the simple Monte-Carlo experiment of Levine & Aguilar (1996), I expect that the galaxies will fill only restricted parts of the available region.

I would venture to predict that the larger ellipticals will move in a way that is close to parallel to the observed ribbon on the fundamental plane, and that the dwarf ellipticals would move in a direction similar to the dwarf “branch”, which is not quite orthogonal to the main ribbon (see figure 1). In a manner analogous to the ensemble of binary stars interacting with other stars, where the hard binaries tend to get harder, and the soft binaries tend to get softer, with a large enough sample of simulations, we may find that the family of elliptical galaxies evolves, due to galaxy-galaxy interaction, so that big galaxies get bigger, and smaller galaxies get smaller.

The other aspect to note, is that once we allow the mass-to-light ratio to change over a transformation, the galaxies as an ensemble are no longer limited to an apparent plane, but can now fill a volume in the space of *observable* parameters (a long, narrow volume, see figure 3). If we presume that there is some correlation between total mass and total light, then perhaps the deviations from “pure” virial equilibrium that people have noted are really a function of variation in the evolution of the mass-to-light ratio and indicative of the relative magnitude of changes effected upon galaxies during their life. The underlying mass might still be in equilibrium, but with a variety of mass-to-light ratios the observed ensemble no longer remains quite planar.

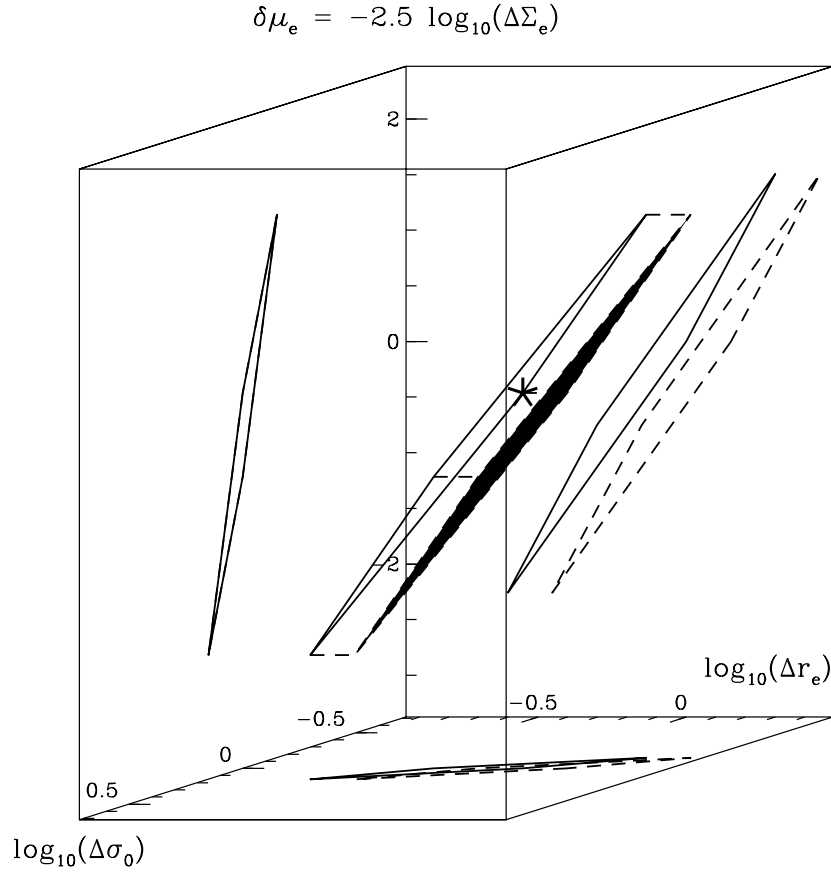
This does not explain how the galaxies reached the Fundamental Plane in the first place, but it may help to show how the populations was separated into



**Figure 2.** The parallelogram which bounds changes of up to  $\Delta M = 1/2$  and  $\Delta E = 1/2$  has been projected into changes in the observable parameters. The open and filled square points show how the parallelogram would be displaced if the global mass-to-light ratio were to change by factors of  $1/2$  and  $2$  respectively. Similarly, the open and filled triangles correspond to changes in BOTH  $\Delta\langle\alpha\rangle$  and  $\Delta\alpha_{r=0}$  of  $1/2$  and  $2$ .

the larger and smaller elliptical branches seen today, and why the Fundamental Plane is not quite flat, but instead shows some thickness. Testing these speculations will depend upon (1) a better understanding of how the mass-to-light ratio varies with position in galaxies, (2) good interaction cross-sections for a wide range of physical interaction parameters, and (3) an idea of what the initial distribution of galaxies was in the observed parameter space. The last of these is really a re-phrasing of the need to understand how the Fundamental Plane formed in the first place.

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**Figure 3.** The change parallelepiped produced by a change of a factor of  $1/2$  in  $M$ ,  $E$  and  $\langle\alpha\rangle$ .  $\Delta\alpha_{r=0} = 1$ . The five pointed dot marks the initial point (0,0,0). The face of the parallelepiped where  $\Delta\langle\alpha\rangle = 1$  is open and drawn in *solid* lines, while that for  $\Delta\langle\alpha\rangle = 1/2$  is shaded, and drawn in *dashed* lines.

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